§ 2.2 Tables of Supersymmetric Deformations

$d=3,\ \mathcal{N}=1$

SCA: $Osp(1|4)$

$\longrightarrow \quad Q \in [1]_{1/2}\ ,\ N_Q = 2$

superconformal unitarity bounds and shortenings:

Name	Primary	Unitarity Bound	Null state
L	$[j]_\Delta\ j\geq 1$	$\Delta > \frac{1}{2}j + 1$	—
L'	$[0]_\Delta$	$\Delta > \frac{1}{2}$	—
A_1	$[j]_\Delta\ j\geq 1$	$\Delta = \frac{1}{2}j + 1$	$[j-1]_{\Delta+\frac{1}{2}}$
A_2'	$[0]_\Delta$	$\Delta = \frac{1}{2}$	$[0]_{\Delta+1}$
B_1	$[0]_\Delta$	$\Delta = 0$	$[1]_{\Delta+1/2}$

Supersymmetric Deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$L'\{\Delta_\mathcal{O} > \frac{1}{2}\}$	$Q^2\mathcal{O} \in \{\Delta > \frac{3}{2}\}$	D-term

$d=3,\ \mathcal{N}=2$

SCA: $\quad osp(2|4)$

$\rightarrow$ R-sym.: $\quad so(2)_R \simeq U(1)_R$

denoted by (r) r-charge

$\rightarrow\ Q \in [1]^{(-1)}_{1/2}, \qquad \bar{Q} \in [1]^{(1)}_{1/2}, \qquad N_Q = 4$

Q-shortening conditions:

Name	Primary	Unitarity Bound	Q Null state
L	$[j]^{(r)}_{\Delta}$	$\Delta > \frac{1}{2}j - r + 1$	—
A_1	$[j]^{(r)}_{\Delta},\ j \geq 1$	$\Delta = \frac{1}{2}j - r + 1$	$[j-1]^{(r-1)}_{\Delta+1/2}$
A_2	$[0]^{(r)}_{\Delta}$	$\Delta = 1 - r$	$[0]^{(r-2)}_{\Delta+1}$
B_1	$[0]^{(r)}_{\Delta}$	$\Delta = -r$	$[1]^{(r-1)}_{\Delta+\frac{1}{2}}$

$\bar{Q}$ - shortening conditions:

Name	Primary	Unitarity Bound	$\bar{Q}$ Null state
$\bar{L}$	$[j]^{(r)}_{\Delta}$	$\Delta > \frac{1}{2}j + r + 1$	—
$\bar{A}_1$	$[j]^{(r)}_{\Delta},\ j \geq 1$	$\Delta = \frac{1}{2}j + r + 1$	$[j-1]^{(r+1)}_{\Delta+1/2}$
$\bar{A}_2$	$[0]^{(r)}_{\Delta}$	$\Delta = 1 + r$	$[0]^{(r+2)}_{\Delta+1}$
$\bar{B}_1$	$[0]^{(r)}_{\Delta}$	$\Delta = r$	$[1]^{(r+1)}_{\Delta+1/2}$

examples:

- $L\bar{B}_1[0]_r^{(r)}$: chiral multiplet
  (annihilated by all $\bar{Q}$ SC)
  $r > \frac{1}{2}$ (consistency of L and $\bar{B}_1$)
- $A_2\bar{B}_1[0]_{1/2}^{(1/2)}$: free scalar field
  $\Delta = \frac{1}{2}$
  (annihilated by Q^2 as well as all $\bar{Q}$ supercharges)
- $A_2\bar{A}_2[0]_1^{(0)}$: conserved flavor current
- $A_1\bar{A}_1[2]_2^{(0)}$: stress-tensor multiplet

Supersymmetric deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$A_2\bar{A}_2 \left\{ \begin{matrix} (0) \\ \Delta_{\mathcal{O}} = 1 \end{matrix} \right\}$	$Q\bar{Q}\mathcal{O} \in \left\{ \begin{matrix} (0) \\ \Delta = 2 \end{matrix} \right\}$	Flavor Current
$L\bar{B}_1 \left\{ \begin{matrix} (r+2),\ r > -\frac{1}{3} \\ \Delta_{\mathcal{O}} = 2+r \end{matrix} \right\}$	$Q^2\mathcal{O} \in \left\{ \begin{matrix} (r),\ r > -\frac{3}{2} \\ \Delta = 3+r > \frac{3}{2} \end{matrix} \right\}$	F-term
$B_1\bar{L} \left\{ \begin{matrix} (r-2),\ r < \frac{3}{2} \\ \Delta_{\mathcal{O}} = 2-r \end{matrix} \right\}$	$\bar{Q}^2\mathcal{O} \in \left\{ \begin{matrix} (r),\ r < \frac{3}{2} \\ \Delta = 3-r > \frac{3}{2} \end{matrix} \right\}$	F-term
$L\bar{L} \left\{ \begin{matrix} (r) \\ \Delta_{\mathcal{O}} > 1+\lvert r\rvert \end{matrix} \right\}$	$Q^2\bar{Q}^2\mathcal{O} \in \left\{ \begin{matrix} (r) \\ \Delta > 3+\lvert r\rvert \end{matrix} \right\}$	D-term

$d=3$, $\mathcal{N}=4$

SCA: $osp(4|4)$

$\rightarrow$ R-sym.: $SO(4)_R \simeq SU(2)_R \times SU(2)'_R$

reps: $(R;R')$, $R, R' \in \mathbb{Z}_{\geq 0}$

($(1;0)$ and $(0;1)$ are left- and right-handed spinors 2 and $2'$ of $SO(4)_R$)

mirror automorphism M: $SU(2)_R \longleftrightarrow SU(2)'_R$

$\rightarrow$ Q-supersymmetries: $Q \in [1]^{(1;1)}_{1/2}$, $N_Q = 8$

Shortening conditions:

Name	Primary	Unitarity Bound	Null State
L	$[j]^{(R;R')}_{\Delta}$	$\Delta > \frac{1}{2}j + \frac{1}{2}(R+R') + 1$	—
A_1	$[j]^{(R;R')}_{\Delta}$, $j \geq 1$	$\Delta = \frac{1}{2}j + \frac{1}{2}(R+R') + 1$	$[j-1]^{(R+1;R'+1)}_{\Delta+1/2}$
A_2	$[0]^{(R;R')}_{\Delta}$	$\Delta = \frac{1}{2}(R+R') + 1$	$[0]^{(R+2;R'+2)}_{\Delta+1}$
B_1	$[0]^{(R;R')}_{\Delta}$	$\Delta = \frac{1}{2}(R+R')$	$[1]^{(R+1;R'+1)}_{\Delta+1/2}$

examples:

- $B_1[0]^{(1;0)}_{1/2}$: free hypermultiplet
- $B_1[0]^{(0;1)}_{1/2}$: free twisted hypermultiplet
- $A_2[0]^{(0;0)}_{1}$: stress-tensor multiplet

$B_1[0]^{(1;0)}_{1/2} \xleftrightarrow{M} B_1[0]^{(0;1)}_{1/2}$ $\qquad$ $A_2[0]^{(0;0)}_{1} \circlearrowleft M$

Supersymmetric deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$B_1\left\{\begin{matrix}(2;0)\\ \Delta_{\mathcal{O}}=1\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(0;2)\\ \Delta=2\end{matrix}\right\}$	Flavor Current (M)
$B_1\left\{\begin{matrix}(0;2)\\ \Delta_{\mathcal{O}}=1\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(2;0)\\ \Delta=2\end{matrix}\right\}$	Flavor Current (M)
$A_2\left\{\begin{matrix}(0;0)\\ \Delta_{\mathcal{O}}=1\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(0;0)\\ \Delta=2\end{matrix}\right\}$	Stress Tensor
$B_1\left\{\begin{matrix}(R+4;0)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}R\end{matrix}\right\}$	$Q^4\mathcal{O}\in\left\{\begin{matrix}(R;0)\\ \Delta=4+\frac{1}{2}R\end{matrix}\right\}$	F-term ($\tilde{M}$)
$B_1\left\{\begin{matrix}(0;R'+4)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}R'\end{matrix}\right\}$	$Q^4\mathcal{O}\in\left\{\begin{matrix}(0;R')\\ \Delta=4+\frac{1}{2}R'\end{matrix}\right\}$	F-term ($\tilde{M}$)
$B_1\left\{\begin{matrix}(R+2;R'+2)\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}(R+R')\end{matrix}\right\}$	$Q^6\mathcal{O}\in\left\{\begin{matrix}(R;R')\\ \Delta=5+\frac{1}{2}(R+R')\end{matrix}\right\}$	—
$L\left\{\begin{matrix}(R;R')\\ \Delta_{\mathcal{O}}>1+\frac{1}{2}(R+R')\end{matrix}\right\}$	$Q^8\mathcal{O}\in\left\{\begin{matrix}(R;R')\\ \Delta>5+\frac{1}{2}(R+R')\end{matrix}\right\}$	D-term

$d=4,\ \mathcal{N}=1$

reps. of Lorentz algebra $so(4) = su(2) \times \overline{su(2)}$:

$$[j;\bar{j}]\ ,\quad j,\bar{j} \in \mathbb{Z}_{\geq 0}$$

SCA: $sl(2,2|1) \longrightarrow u(1)_R$ sym.

supercharges: $Q \in [1;0]^{(-1)}_{1/2}$, $\bar{Q} \in [0;1]^{(1)}_{1/2}$, $N_Q = 4$

Q-shortening conditions :

Name	Primary	Unitarity Bound	Q Null State
L	$[j;\bar{j}]^{(r)}_{\Delta}$	$\Delta > 2 + j - \frac{3}{2}r$	—
A_1	$[j;\bar{j}]^{(r)}_{\Delta}$, $j \geq 1$	$\Delta = 2 + j - \frac{3}{2}r$	$[j-1;\bar{j}]^{(r-1)}_{\Delta+1/2}$
A_2	$[0;\bar{j}]^{(r)}_{\Delta}$	$\Delta = 2 - \frac{3}{2}r$	$[0;\bar{j}]^{(r-2)}_{\Delta+1}$
B_1	$[0;\bar{j}]^{(r)}_{\Delta}$	$\Delta = -\frac{3}{2}r$	$[1;\bar{j}]^{(r-1)}_{\Delta+1/2}$

$\bar{Q}$-shortening conditions:

Name	Primary	Unitarity Bound	$\bar{Q}$ Null State
$\bar{L}$	$[j;\bar{j}]^{(r)}_{\Delta}$	$\Delta > 2 + \bar{j} + \frac{3}{2}r$	—
$\bar{A}_1$	$[j;\bar{j}]^{(r)}_{\Delta}$, $\bar{j} \geq 1$	$\Delta = 2 + \bar{j} + \frac{3}{2}r$	$[j;\bar{j}-1]^{(r+1)}_{\Delta+1/2}$
$\bar{A}_2$	$[j;0]^{(r)}_{\Delta}$	$\Delta = 2 + \frac{3}{2}r$	$[j;0]^{(r+2)}_{\Delta+1}$
$\bar{B}_1$	$[j;0]^{(r)}_{\Delta}$	$\Delta = \frac{3}{2}r$	$[j;1]^{(r+1)}_{\Delta+1/2}$

examples:

- $L\bar{L}[j;\bar{j}]^{(r)}_{\Delta}$: long multiplet
- $L\bar{B}_1[j;0]^{(r)}_{3r/2}$: generic chiral multiplet (annihilated by all $\bar{Q}$-SCs)
- $A_2\bar{B}_1[0;0]^{(2/3)}_{1}$: free scalar field with $j=0, \Delta=1$
- $A_1\bar{B}_1[1;0]^{(1)}_{3/2}$: free vector multiplet
- $A_2\bar{A}_2[0;0]^{(0)}_{2}$: Conserved flavor currents
- $A_1\bar{A}_1[1;1]^{(0)}_{3}$: stress-tensor multiplet

Supersymmetric Deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$L\bar{B}_1\left\{\begin{matrix}(r+2), \; r>-\frac{4}{3} \\ \Delta_{\mathcal{O}}=3+\frac{3}{2}r\end{matrix}\right\}$	$Q^2\mathcal{O}\in\left\{\begin{matrix}(r), \; r>-\frac{4}{3} \\ \Delta=4+\frac{3}{2}r>2\end{matrix}\right\}$	F-term
$B_1\bar{L}\left\{\begin{matrix}(r-2), \; r<\frac{4}{3} \\ \Delta_{\mathcal{O}}=3-\frac{3}{2}r\end{matrix}\right\}$	$\bar{Q}^2\mathcal{O}\in\left\{\begin{matrix}(r), \; r<\frac{4}{3} \\ \Delta=4-\frac{3}{2}r>2\end{matrix}\right\}$	F-term
$L\bar{L}\left\{\begin{matrix}(r) \\ \Delta_{\mathcal{O}}>2+\frac{3}{2}\lvert r\rvert\end{matrix}\right\}$	$Q^2\bar{Q}^2\mathcal{O}\in\left\{\begin{matrix}(r) \\ \Delta>4+\frac{3}{2}\lvert r\rvert\end{matrix}\right\}$	D-term

$d=4$, $\mathcal{N}=2$

SCA: $SU(2,2|2)$

$\to$ R-sym. : $SU(2)_R \times U(1)_R$

$(R; \quad r)$, $R \in \mathbb{Z}_{\geq 0}$, $r \in \mathbb{R}$

$\to$ $Q \in [1;0]^{(1;-1)}_{1/2}$, $\bar{Q} \in [0;1]^{(1;1)}_{1/2}$, $N_Q = 8$

Q-shortening conditions:

Name	Primary	Unitarity Bound	Q Null state
L	$[j;\bar{j}]^{(R;r)}_{\Delta}$	$\Delta > 2 + j + R - \frac{1}{2}r$	—
A_1	$[j;\bar{j}]^{(R;r)}_{\Delta}$, $j \geq 1$	$\Delta = 2 + j + R - \frac{1}{2}r$	$[j-1;\bar{j}]^{(R+1;r-1)}_{\Delta+1/2}$
A_2	$[0;\bar{j}]^{(R;r)}_{\Delta}$	$\Delta = 2 + R - \frac{1}{2}r$	$[0;\bar{j}]^{(R+2;r-2)}_{\Delta+1}$
B_1	$[0;\bar{j}]^{(R;r)}_{\Delta}$	$\Delta = R - \frac{1}{2}r$	$[1;\bar{j}]^{(R+1;r-1)}_{\Delta+1/2}$

$\bar{Q}$-shortening conditions:

Name	Primary	Unitarity Bound	Q Null state
$\bar{L}$	$[j;\bar{j}]^{(R;r)}_{\Delta}$	$\Delta > 2 + \bar{j} + R + \frac{1}{2}r$	—
$\bar{A}_1$	$[j;\bar{j}]^{(R;r)}_{\Delta}$, $\bar{j} \geq 1$	$\Delta = 2 + \bar{j} + R + \frac{1}{2}r$	$[j;\bar{j}-1]^{(R+1;r+1)}_{\Delta+1/2}$
$\bar{A}_2$	$[j;0]^{(R;r)}_{\Delta}$	$\Delta = 2 + R + \frac{1}{2}r$	$[j;0]^{(R+2;r+2)}_{\Delta+1}$
$\bar{B}_1$	$[j;0]^{(R;r)}_{\Delta}$	$\Delta = R + \frac{1}{2}r$	$[j;1]^{(R+1;r+1)}_{\Delta+1/2}$

examples:

- $L\bar{B}_1[0;0]^{(0;r)}_{r/2}$: chiral multiplet (annihilated by all $\bar{Q}$ supercharges)
- $B_1\bar{B}_1\,[0;0]^{(2;0)}_{2}$: conserved flavor current

Supersymmetric Deformations:

Primary $\mathcal{O}$	Deformation $\delta\mathcal{L}$	Comments
$B_1\bar{B}_1\begin{Bmatrix}(2;0)\\ \Delta_{\mathcal{O}}=2\end{Bmatrix}$	$Q^2\mathcal{O}\oplus\bar{Q}^2\mathcal{O}\in\begin{Bmatrix}(0;-2)\oplus(0;2)\\ \Delta=3\end{Bmatrix}$	Flavor Current
$B_1\bar{B}_1\begin{Bmatrix}(R+4;0)\\ \Delta_{\mathcal{O}}=4+R\end{Bmatrix}$	$Q^2\bar{Q}^2\mathcal{O}\in\begin{Bmatrix}(R;0)\\ \Delta=6+R\end{Bmatrix}$	F-term
$L\bar{B}_1\begin{Bmatrix}(0;r+4),\ r>-2\\ \Delta_{\mathcal{O}}=2+\frac{1}{2}r\end{Bmatrix}$	$Q^4\mathcal{O}\in\begin{Bmatrix}(0;r),\ r>-2\\ \Delta=4+\frac{1}{2}r>3\end{Bmatrix}$	F-term
$L_1\bar{L}\begin{Bmatrix}(0;r-4),\ r<2\\ \Delta_{\mathcal{O}}=2-\frac{1}{2}r\end{Bmatrix}$	$\bar{Q}^4\mathcal{O}\in\begin{Bmatrix}(0;r),\ r<2\\ \Delta=4-\frac{1}{2}r>3\end{Bmatrix}$	F-term
$L\bar{B}_1\begin{Bmatrix}(R+2;r+2),\ r>0\\ \Delta_{\mathcal{O}}=3+R+\frac{1}{2}r\end{Bmatrix}$	$Q^4\bar{Q}^2\mathcal{O}\in\begin{Bmatrix}(R;r),\ r>0\\ \Delta=6+R+\frac{1}{2}r>6+R\end{Bmatrix}$	
$B_1\bar{L}\begin{Bmatrix}(R+2;r-2),\ r<0\\ \Delta_{\mathcal{O}}=3+R-\frac{1}{2}r\end{Bmatrix}$	$Q^2\bar{Q}^4\mathcal{O}\in\begin{Bmatrix}(R;r)\\ \Delta=6+R-\frac{1}{2}r>6+R\end{Bmatrix}$	
$L\bar{L}\begin{Bmatrix}(R;r)\\ \Delta_{\mathcal{O}}>2+R+\frac{1}{2}\lvert r\rvert\end{Bmatrix}$	$Q^4\bar{Q}^4\mathcal{O}\in\begin{Bmatrix}(R;r)\\ \Delta>6+R+\frac{1}{2}\lvert r\rvert\end{Bmatrix}$	D-term